The Self-reconfiguring Robotic Molecule: Design and Control Algorithms

Keith Kotay, *Dartmouth College, Hanover, NH, USA*
Daniela Rus, *Dartmouth College, Hanover, NH, USA*
Marsette Vona, *Dartmouth College, Hanover, NH, USA*
Craig McGray, *Dartmouth College, Hanover, NH, USA*

We discuss a robotic module called a Molecule. *Molecules can be the basis for building self-reconfiguring robots. They support multiple modalities of locomotion and manipulation. We describe the design, functionality, and control of the Molecule. We show how a set of Molecules can aggregate as active three-dimensional structures that can move and change shape. We show an efficient algorithm for planning the trajectory of Molecules for self-reconfiguration. Finally, we discuss our Molecule experiments.*

1 Introduction

Our vision is to create versatile robots by using self-reconfiguration: hundreds of small modules will autonomously organize and reorganize as geometric structures to best fit the terrain on which the robot has to move, the shape of the object the robot has to manipulate, or the sensing needs for the given task. Large collections of small robots will actively organize as the most optimal geometric structure to perform useful coordinated work.

A self-reconfiguring robot consists of a set of identical modules that can dynamically and autonomously reconfigure in a variety of shapes, to best fit the terrain, environment, and task. Self-reconfiguration leads to versatile robots that can support multiple modalities of locomotion and manipulation. For example, a self-reconfiguring robot can aggregate as a snake to traverse a tunnel and then reconfigure as a six-legged robot to traverse rough terrain, such as a Lunar surface, and change shape and gait again to climb stairs and enter a building.

We build on the ground-breaking work of [14, 17, 12, 5] who introduced the first robot systems capable of self-reconfiguration. We have designed a small and simple robotic module we call a *Molecule* capable of self-reconfiguration in three dimensions. The Molecule (see Figure 1) is capable of independent movement on a substrate of identical Molecules, including straight-line traversal and 90 degree convex and concave transitions to adjacent surfaces. In this paper we describe the motion control for these operations and we show a geometric approach to specifying and planning molecular motion on a substrate of Molecules in $O(n)$ time, where n is the number of Molecules in the substrate.

The Molecule is a 4 degree-of-freedom, small-scale module capable of aggregating with other identical modules to form three-dimensional dynamic structures. The Molecule consists of two *atoms* connected by a right-angle rigid bond (see Figure 2.) Each atom has 5 inter-Molecule connectors and two degrees of freedom. One degree of freedom allows rotation about one connector. The second degree-of-freedom allows rotation of the atom about the bond. Detailed descriptions of the design and control of the Molecule can be found in [8].

A set of such Molecules can self-aggregate as arbitrary three-dimensional structures. We show this result by demonstrating several different tilings of the plane with the Molecule. The tilings can be stacked so that the resulting structures are three-dimensional. A structure made of Molecules can self-reconfigure by using the basic Molecule motions. For example, a structure made of Molecules can climb a set of stairs by self-reconfiguration as follows (see Figure 5). Linear motions are used to place the structure at the bottom of the staircase. Concave transitions and linear motions enable Molecules to climb on top of each other, composing a tower whose height equals the height of the

step. The tower can then be used by other Molecules to climb up and move onto the step. This process is then repeated for each step.

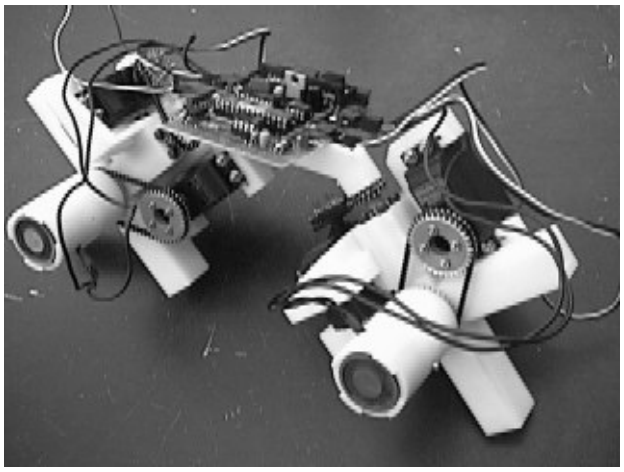

Figure 1: *The robotic Molecule. The Molecule is composed of two atoms, connected by an right-angle rigid bond. The Molecule has 4 degrees of freedom: two rotational degrees of freedom about the bond and one rotational degree of freedom per atom about a single inter-Molecule connector. The connectors have been implemented with electromagnets.*

In our lab we have prorotyped a Molecule and experimented with its basic motion capabilities. The Molecule performed linear motion and transitions. These experiments validated our design. We plan to build 16 modules to test our theory of self-reconfiguration.

This paper is organized as follows. We continue with a summary of related work. We then describe the mechanical design of the Molecule and the fabrication process. We address how this Molecule can form arbitrary three dimensional structures. Finally, we show a geometric approach to computing the motion of individual Molecules on a substrate of n other Molecules in $O(n)$ time, by constructing a reachability graph and searching this graph for a path. This graph can be used to extract a motion plan in the Molecule control space.

2 Related work

Our work draws on previous experiences with navigation algorithms [10], designing self-organizing robots, and designing minimalist robot systems [4].

Related work in designing modular robots includes [15, 16, 13, 6]. In [15] a method for designing various robotic arms with different reachability properties out of the same set of 7 modules is proposed. The mechanical design algorithm is implemented as simulated annealing that starts with a random mechanical design and converges to the design with desired reachability properties. The modules are assembled by hand as the computed shape. Our work is different in that our modules could self-aggregate (without human intervention) and the planning phase is of a task-directed, geometric nature. Our modular self-reconfiguring robots are aggregated according to task and we can view reachability as a specific kind of task.

Related work in self-organizing robots includes robots in which modules are reconfigurable using external intervention [3]. In [5] a cellular robotic system is proposed to coordinate a set of specialized modules. [1] describe a theoretical framework for counting the number of unique configurations realizable from a set of modules and joints, without considering implementation issues. [17] studies multiple modes of locomotion that are achieved by composing a few basic elements in different ways. [12, 18] consider a system of modules that can achieve planar motion by walking over each other due to changes in the polarity of magnetic fields. [14] describes metamorphic robots that can aggregate as stationary two-dimensional structures with varying geometry and that implement planar locomotion.

Our self-reconfigurable robots [8, 7] are different from metamorphic robots in their design and functionality. The structures built from self-organizing Molecules and Inchworms are three dimensional, can move along any axis in a three dimensional space, and have motion autonomy relative to a three-dimensional world.

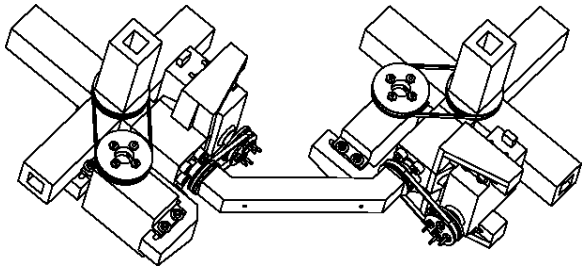

Figure 2: *The CAD model of the Molecule. The Molecule consists of two atoms. A rigid bond that connects the two atoms so that their relative orientation is 90 degrees. The figure shows four actuators, one per degree of freedom. The stalks in the figure represent inter-Molecule connection points. The connection betwee*

3 The Design of the Molecule

3.1 Concept

The initial goal for our Molecule design was to create a regular solid shape which could be closely packed in 3-D space, and which could move about by attaching itself to a substrate of similar units. Since a cube is the simplest regular solid which fully packs in 3-D space, we decided on a cube-shaped design similar to an atom in our current design (Figure 2). This design had inter-unit connectors on each face of the cube, with a rotational degree of freedom about each connector. This design was not capable of independent movement however, two cooperating connected units were necessary for movement. Furthermore, movement required that a unit be able to support the weight of its partner. Because each unit required 6 rotational degrees of freedom plus the weight of the connection mechanisms, we felt it would be very difficult to create a unit which was strong enough to lift another unit.

Consequently, we attempted to reduce the number of degrees of freedom in our unit without severely reducing its capabilities. The result is our robotic Molecule (see Figure 2). It consists of two atoms linked by a rigid 90 degree connection we call the *bond*. Each atom has five inter-Molecule connection points and two degrees of freedom. One degree of freedom allows the atom to rotate 180 degrees relative to its bond connection, and the other degree of freedom allows the atom (and therefore the entire Molecule) to rotate 180 degrees relative to one of the inter-Molecule connectors (the rotational degree of freedom cannot be about the connector opposite the bond connection). This design is capable of independent movement on a substrate of identical Molecules, including straight-line traversal and 90 degree concave and convex transitions to adjacent surfaces. However, the L-shaped design cannot be as closely packed in 3-D space as a cube-shaped design (see section 7).

3.2 Implementation

Our current design uses R/C servomotors for the rotational degrees of freedom and electromagnets for the inter-Molecule connectors. The prototype Molecule uses only a single connector on each atom. This connector is attached to the non-bond rotational degree of freedom as described above. The rotating connection points on each atom are the only connection points required for Molecule motion. The other connection points are used for attachment to other Molecules to create stable 3-D structures. A Molecule to Molecule connection requires the connecting electromagnets to be oppositely polarized so that the electromagnetic fields will attract. The 1" electromagnets used on our prototype are sufficiently powerful to support an entire Molecule. To prevent the rotation of one electromagnet with respect to another, we developed interlocking sheaths which encircle the contacting faces of the electromagnets.

Each Molecule also contains a microprocessor and the circuitry needed to control the servomotors and electromagnets. The microprocessor performs low-level control of the hardware but currently the high-level control of the Molecule takes place off-board in a workstation. The Molecule communicates with the workstation using an RS-485 serial connection.

3.3 Fabrication

We have built a prototype Molecule to demonstrate the feasibility of our approach. The parts cost of the pro-

totype Molecule is about $1000, and it takes approximately three days to fabricate and assemble all parts. We have utilized several technologies to reduce the design iteration time, improve the mechanical precision, and reduce the cost of the prototype[1]. Our design begins with a fully detailed and dimensioned 3-D CAD model of the prototype developed with Pro/Engineer 18.0 from Parametric Technolgy Corp. We then fabricate most of the structure using an FDM1600 Fused Deposition Modeling (FDM) Rapid Prototyping machine manufactured by Stratasys, Inc. This machine and its associated software converts the data from our CAD model directly into high-strength, light weight ABS plastic parts. These ABS parts are assembled together with the electromagnets, the servos, and a few other parts to create a full Molecule.

4 Aggregating Three-dimensional Structures

The Molecule can connect with other identical Molecules to create dynamic three dimensional geometric structures. Figure 3 shows an example of creating a wall by packing Molecules. It is possible to create arbitrary three-dimensional geometric shapes consistent with the Molecular structure. These three-dimensional shapes can only have surfaces that meet at angles of 90 degrees.

Molecules can be placed next to each other without explicit connections. To increase the robustness of the resulting structures, we insist that each Molecule in a structure be connected to at least one other Molecule. One can imagine stringing Molecules together subject to this constraint in an arbitrary fashion. An interesting question is, what the class of three-dimensional structures that can be created using the Molecule? We propose examining this question by defining planar tilings using the Molecule.

Consider a possible tiling of the plane using the Molecule. It is interesting to observe that if we place an xy grid whose size equals the atom size on the plane, each Molecule will occupy three cells in this grid, arranged as an "L" shape. We would like to find ways of covering every cell in such a planar grid with the Molecule. Figure 4(Left) shows an example of a possible tiling that covers every cell in the grid. The generator for the tiling consists of a "string" of two Molecules. Other tilings are also possible. Figures 4(Center) and 4(Right) show two other generators and the resulting tilings. Note that in all three figures the Molecules are simplified by omitting the actuators. In Section 7 we will show how such tilings can be used to generate dynamic structures made out of Molecules that can travel along any direction. In this section we show how tilings help with constructing arbitrary three dimensional objects out of Molecules.

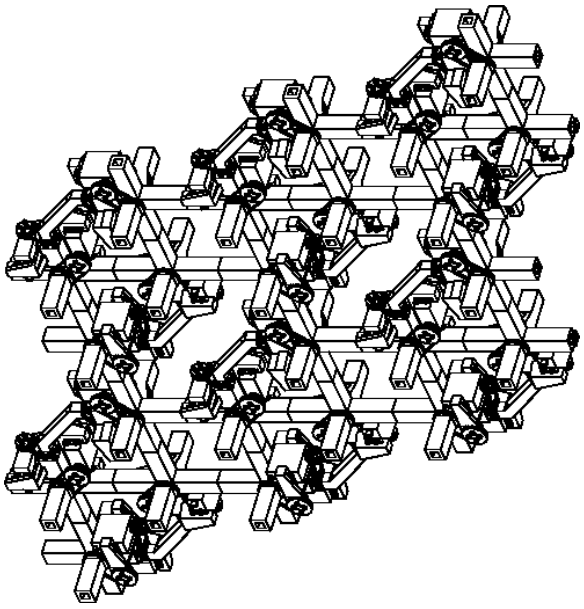

Figure 3: *This figure shows a planar wall created by tiling with the Molecule. This figure details each Molecule to illustrate the location of its connectors and the actuators.*

To show this result, we will use a cell-decomposition method. We define a cell decomposition of a three-dimentional object O to be the division of the object into a union of disjoint cells. An l-grid cell decomposition is a cell decomposition into equal cells where each cell is a square and the side of the square is l. An

[1]We thank Brian Locke and the Thayer School of Engineering for their help in this process.

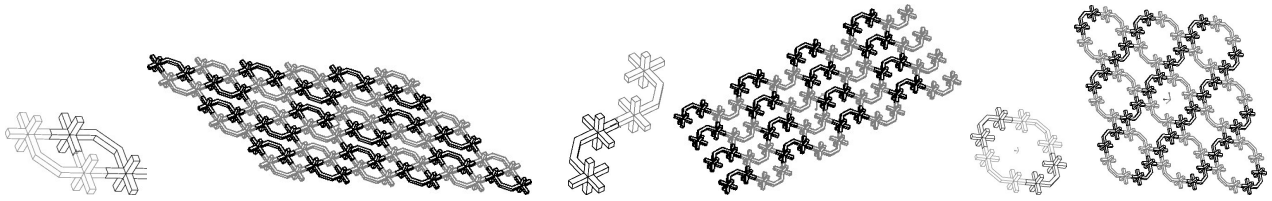

Figure 4: *This figure shows three planar tilings using the Molecule. The "pair" generator on the left is used to create the leftmost tiling. The "string" generator in the middle is used to create the middle tiling. The "ring" generator on the right is used to create the rightmost tiling. The use of color (black or gray) is to visualize better where each individual Molecule is locatet. Lated in Section 7 we show that this particular color encoding of Molecular structures has interesting geometric properties with respect to planning.*

l-grid approximates a three-dimensional object with a Manhattan object[2] that consists of a union of identical cubes.

Theorem 1 *Any connected three-dimensional Manhattan object that allows an l-grid cell decomposition where l, the size of the grid, is the size of a pair tile can be created using Molecule robots.*

Proof: It is possible to place an xyz grid of size equal to the tile size on the object. Since pair tiles pack tightly in the plane, we can construct each xy section of the object by selecting an appropriate planar shape. Since pair tiles are flat (i.e. all atoms are coplanar) we can stack these tiles on top of one another in a tightly packed fashion. So, we can construct each xy section of the object by selecting an appropriate planar shape. These layers can be stacked to obtain the desired object. □

Corollary 2 *Any three-dimensional structure that allows a cell decomposition where the size of the grid is the size of an individual Atom, can be approximated using Molecule robots, with an approximation error of at most two cells for every cell on the xy exterior of the structure.*

Proof: Begin by outlining each xy slice of the desired Manhattan Object. Stack pair tiles so that all cells in each slice of the object are completely covered. Now, remove any Molecules that do not cover at least one cell in the desired object. Among the remaining Molecules, there may be some that cover one or two cells that are not in the desired structure. These Molecules, must, however, cover at least one cell in the desired structure or they would have been removed already. Furthermore, since this cell is adjacent to a cell in the same plane that is not in the desired structure, it must be on the xy exterior. So, for each cell on the xy exterior of the desired structure, there can be at most two cells in the actual structure that are not in the desired structure. □

Note that Theorem 1 and Corollary 2 give a method for constructing objects out of Molecules and a measure for quantifying the approximation of the construction only when the size of the object is large relative to the size of an individual Molecule. One can imagine building these Molecules at the MEMS scale. Such a construction would make possible the construction of objects with high-resolution surfaces.

A caveat of Theorem 1 and its corollary is that the weight of the object can be supported by its molecular layers. Since the body of our Molecule is currently made out of plastic, this places some restrictions over how many can be stacked on top of each other. We are currently investigating this question.

[2] A Manhattan object has all its surfaces either parallel or perpendicular to one another.

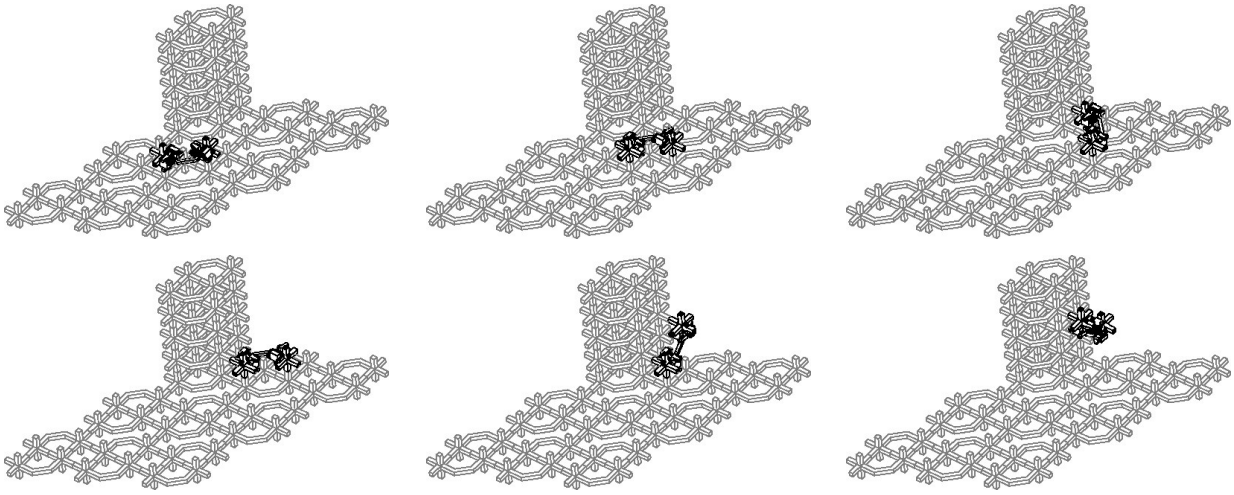

Figure 5: *This figure shows a Molecule walking on a structure composed of identical Molecules. The structure, shown in grey, consists of a horizontal plane and a vertical tower. The Molecule, shown in black, starts on the plane (top, left), rotates 180 degrees about the connector of its right atom to the plane (top, center), rotates 90 degrees about the connector of its right atom to the plane (top, right), rotates 90 degrees about the connector of its left atom to the plane (bottom, left), rotates vertically 90 degrees about the bond connector of the top atom to transition and make contact with the tower (bottom, center), and finally it rotates 180 degrees about the connector of its top atom to the wall to traverse the tower.*

5 Molecule Motion Control

In the previous section we described the class of three-dimensional objects that can be built out of Molecules. Objects belonging to this class can metamorphose into one another by using the Molecule degrees of freedom. In this section we focus on the Molecule motion control for moving an individual Molecule on a substrate of other Molecules. In Section 6 we give a geometric approach to planning Molecule trajectories on a substrate of other Molecules. In [9, 11] we present other planning bounds and algorithms for controlling the global motion of a structure made of Molecules.

Molecule motion is controlled by attaching an atom to some other Molecule and actuating one or more of the four degrees of freedom of the Molecule. Because atoms can only occupy points in a cubic lattice, only actuator positions which rotate an atom (or the entire Molecule) by 90 degrees are valid for attachment. Therefore, in theory, only discrete 90 degree actuator positions are required. However, in practice, other actuator positions are used to compensate for Molecule misalignment due to mechanical tolerances.

Our Molecule design is capable of linear walking on a planar lattice of Molecules, and concave and convex transitions to planar lattices of Molecules oriented at 90 degrees to the original surface. We describe the algorithms for these motions in the following sections.

5.1 Linear walking

Figure 6 shows the three stages in a linear walk. The arrows represent the positions of the connection point actuators. The atom with the bold outline is connected to the underlying lattice represented by the checkerboard pattern. Figure 6(Left) represents the initial state of the Molecule. Figure 6(Center) shows the result after the Molecule has pivoted clockwise 90 degrees about the connected atom (the atom with the gray arrow) and has swapped atom connections. Note that the atom designated by the black arrow also performed a 90-degree rotation of its connection point while it was being moved to its new position (if the connection point

had not rotated with respect to the atom it would be pointing right instead of up). This was done to compensate for the rotation required by the next move. Figure 6(Right) shows the result after the Molecule has rotated counterclockwise 90 degrees about the connected atom (the atom with the black arrow) and has swapped atom connections. Note again that the moving atom has also performed a 90 degree rotation of its connection point. Since Figure 6(Right) shows the Molecule in the same pose as Figure 6(Left) and the Molecule has moved forward one white square, this algorithm can repeated as often as necessary to move the Molecule a given amount in a straight line, assuming that the underlying lattice exists to support the Molecule.

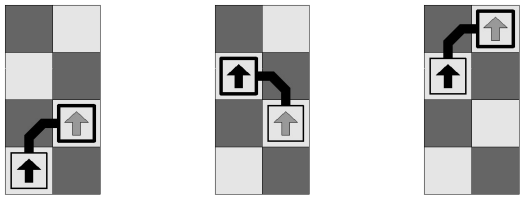

Figure 6: *A linear walk sequence.*

5.2 Concave transition

Figure 7 shows the sequence of Molecule moves necessary to perform a concave transition. The atoms are represented by large blue cubes and the small yellow, cyan, and purple cubes represent the inter-Molecule connectors. Yellow connectors can rotate, the others cannot. The checkered surface represents a lattice of Molecule connectors. Figure 7(Left) shows the Molecule in the initial position. The atom nearest to the wall is attached to the floor surface. Figure 7(Second from left) shows the pose after a 90-degree rotation about the bond connection of the attached atom. This causes the unattached atom to be suspended in midair one atom position to the left and one atom position above its original position. Figure 7(Second from right) shows the pose after a 90-degree rotation about the inter-Molecule connector of the attached atom. The result of this motion is to bring the unattached atom into contact with the wall. At this point the atoms swap connections so that the previously unattached atom is now attached, and the previously attached atom is unattached. Figure 7(Right) shows the final stage of the transition in which the lower atom is rotated to place its inter-Molecule connector on the wall. From this point the Molecule can execute a linear walk to climb the wall.

5.3 Convex transition

Figure 8 shows the sequence of moves necessary to perform a convex transition. Figure 8(Upper Left) shows the initial position of the Molecule. The atom with the cyan non-rotating inter-Molecule connectors is attached to the checkered surface by its rotating (yellow) inter-Molecule connector. The first rotation is about this connector and it causes the unattached atom to swing out over the convex edge. The next rotation about the bond connection of the unattached atom positions the rotating connector so that it faces the vertical surface. A rotation about the bond connection of the attached atom then moves the unattached atom into contact with the vertical surface below the attached atom. Atom connections are then swapped so that the atom with the purple non-rotating connectors is now attached the the lattice, and the atom with the cyan non-rotating connectors is free to move. The next three rotations move the newly unattached atom into free space above the vertical surface, position the rotating connector of the unattached atom so that it faces the vertical surface, and move the unattached atom into contact with the vertical surface. At this point the Molecule can traverse the vertical surface using the linear walk algorithm.

6 Molecule Trajectory Control

The Molecule motion algorithms are specified at the level of very basic control, as a combination of individual rotations. For more global motions, especially when the robotic system contains large numbers of Molecules, this approach is too low-level and difficult to reason about, manage, and program. When multiple Molecules move in parallel as a distributed sys-

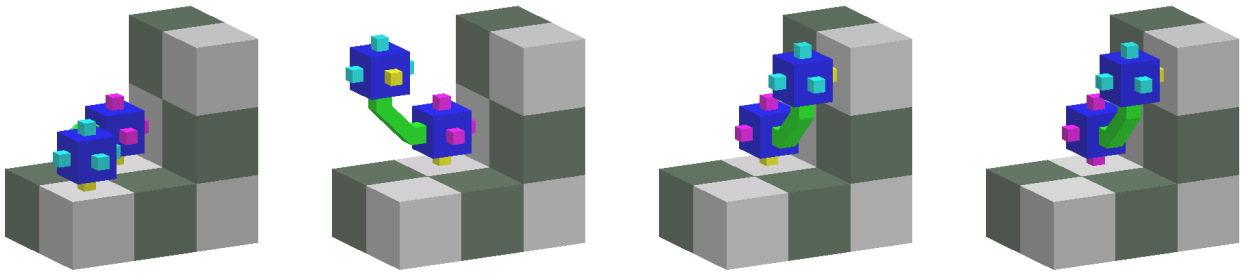

Figure 7: *A concave transition sequence. Rotating connectors are yellow, non-rotating connectors are cyan on one atom and purple on the other atom. The bond is green. The checkered surface represents a lattice of Molecule connectors.*

tem, individual motion management becomes impossible. For example, over 15 individual rotations are currently used to implement one primitive in the global (planar) walk of a two module or four module structure [9].

In this section we show a geometric formulation of Molecule motions that reduces the trajectory planning problem for individual Molecules to graph search.

6.1 A Graph Model

Suppose S is a connected molecular structure composed of k Molecules arranged as an arbitrary three-dimensional structure. Because Molecules connect to one another either in the plane or by stacking up, S is a Manhattan object. Let xyz be a system of coordinates that aligns the axes with the main directions of S. The unit in this system of coordinates is the length of an atom.

Consider the following coloring scheme for the Molecule structure. For every atom, let s be the sum of its x, y, and z coordinates. If s is even, assign the atom the color *blue*. If s is odd, assign the atom the color *red*. This coloring scheme defines a unique partition for S with the following properties:

1. two atoms that belong to the same Molecule have the same color (because they must be placed along some diagonal)
2. two adjacent (connected) atoms that belong to different Molecules have different color (because there is a difference of one unit in one coordinate only)

For example, if S is a plane, this coloring scheme defines a checkered surface.

Now, consider the set of planar faces that defines the exterior of S. Let F be such a face.

Theorem 3 *A blue Molecule that rests on F has to start on top of a red Molecule and it can reach any red location on F to which there is a continuous diagonal sequence of red atoms. A red Molecule that rests on F has to start on top of a blue Molecule and it can reach any blue location on F to which there is a continuous diagonal sequence of blue atoms.*

Proof: The coloring scheme justifies the initial state. The planar walking algorithm described in Section 5 is capable of traversing any diagonal and of crossing from one diagonal to any perpendicular diagonal. □

An interesting consequence of Theorem 3 follows when we observe that in order for a Molecule to move within a structure its first step is to climb on top of a Molecule colored with a different color.

Corollary 4 *Molecules do not change partitions.*

Theorem 3 and Corollary 4 suggest a graph representation for computing reachability on the face F. Let $G_b = (V_r, E_r)$ be the undirected graph that captures the reachability set of blue Molecules on F. Let

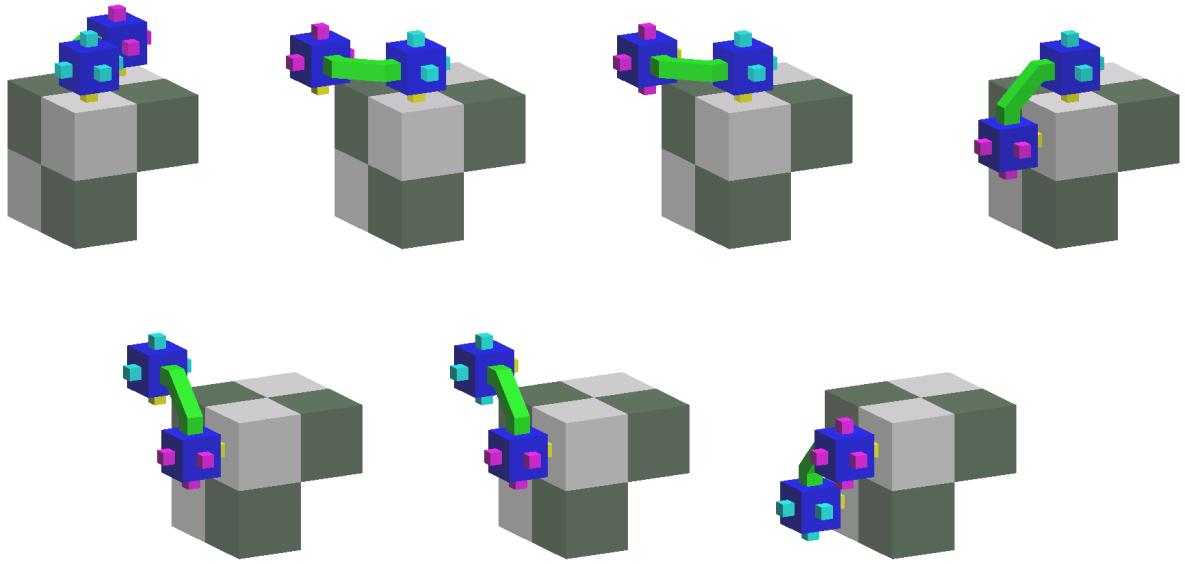

Figure 8: *A convex transition sequence.*

$G_r = (V_b, E_b)$ be the undirected graph that captures the reachability set of red Molecules on F. We will construct these graphs as follows. Without loss of generality, we will illustrate the algorithm for $G_r = (V_b, E_b)$.

1. the vertex set V_b consists of all blue atoms on F
2. the edge set E_b connects any two vertices adjacent along diagonals

It is straightforward to see that Molecule trajectories correspond to paths in these graphs.

6.2 Trajectory Planning

In this section we show how to specify geometrically a three-dimensional trajectory for a Molecule.

Our first observation is that Molecules move on any exterior face of the structure. Within each face, the motion is captured by a graph as described in Section 6.1. Moving from one location to another within this face reduces to searching this graph.

The face graph construction can be generalized to capture any trajectory of the Molecule along the exterior surface of the structure as the graphs $G_b = (V_r, E_r)$ and $G_r = (V_b, E_b)$ as follows. Intuitively, the vertex set is the union of the vertex sets for each face. The edge set is the union of the edge sets for each face, plus transition edges. For any pair of adjacent faces a transition edge connects an atom from one face to an atom on the other face when it is possible to place a Molecule on top of those two atoms. The number of transition edges from any individual atom of one face to the adjacent face is constant, because each Molecule has only 4 degrees of freedom. It is easy to see that the $|E_r| = O(|V_r|)$ and $|E_b| = O(|V_b|)$.

The following result follows.

Theorem 5 *Let S be a Molecular structure of k Molecules. Let M be a Molecule on the exterior of this structure. Let I be an initial location and F be a final location for M. The shortest path from I to G can be computed in $O(k)$.*

Proof: The motion plan from I to G can be obtained

by by constructing the reachability graph for M and searching the graph for the shortest path in the number of edges. The reachability graph $G = (V, E)$ for this structure can be computed in $O(k)$. The size of the edge set $|E| = O(k)$. A breadth-first-search will compute the shortest path, or find that no path exists in $O(k)$.

To compute an actual motion plan for the Molecule, an expression in the control space of individual Molecules[3] has to be synthesized from the graph. To do this, the construction of the graph is augmented with the following step. For each computed edge, a label that indicates the type of rotation required to make the step can be determined straightforwardly by examining local geometry. The motion plan consists of an expression in the language motion obtained by concatenating the graph labels during the traversal from I to G. □

7 Mobility and Gaits

Molecules can walk on three dimensional structures by using the trajectory control and planning algorithms descried in Section 6. Figure 5 shows an example in which the Molecule travels to the base of a tower and climbs on the tower. Note that in this example, the planar surface is composed with a "pair" tiling and the tower is created by stacking "pair" tilings. In general, the Molecule motion follows a pattern that is dictated by the geometry of the underlying Molecule substrate. In this section we examine several different Molecule gaits and their properties for the three tilings described in Section 4.

7.1 Moving along a straight-line trajectory

The pattern of individual rotations that achieves a given transition in the x (or y) direction is called a *gait*. Figure 4 shows three different tilings for the base plane. Note that each generator can be partitioned in two sets and the Molecules in each set can be colored either red or blue. We choose this color encoding because it helps us visualize better where each Molecule is located. We will also show that this particular color encoding has interesting properties with respect to planning.

Suppose there is an infinite plane generated by one of the given tiles. Suppose we wish for Molecule M composed of atoms A_1 and A_2 to effect a unit translation in the x direction. We add a system of coordinates in the following way. Each atom consists of 4 connectors oriented at 90 degree intervals. Let the x axis be defined by two colinear connectors and let the y axis be defined by the other two colinear connectors. The two axies defined in this manner are orthogonal. The origin of this system of coordinates can be anywhere in the plane. Without loss of generality, suppose the origin is the lowest left-most atom in the tiling. Given an integer d, the following algorithms will generate translations by 1 unit in the x and/or y direction of the tiled plane. Each algorithm can be iterated d times to generate translations by d units.

If the planar surface is tiled with "pairs", M will perform a rotation of 180 degrees about A_1, followed by a rotation of -90 degrees about A_2, followed by a rotation of 90 degrees about A_1. Translations in the y direction are similar.

Theorem 6 *A Molecule that starts on top of a black Molecule in a pair tiling can reach all the other black locations and none of the gray locations. A Molecule that starts on top of a gray Molecule in a pair tiling can reach all the other gray Molecules and none of the black Molecules.*

Proof: A Molecule can only start on top of another Molecule so both its atoms rest on the same color Molecules in the initial configuration. Without loss of generality, suppose the Molecule starts on top of a black Molecule. Each 180 degree rotation or 90 degree rotation allows one of the atoms of the Molecule to reach another black atom in the tiling. Rotations by 180 degrees allow the Molecule to reach other black Molecules along the diagonal. Since the Molecule can move by 1 unit to the next black position on the x axis and it can move by 1 unit to the next black position on the y axis, by composing these moves the Molecule can reach any black position in the tiling. □

[3]The control space of a Molecule consists of the four rotations each Molecule is capable of executing.

Similar results hold for "ring" and "string" tiles. Note that for "ring" tilings, translations in the x direction are done with a rotation by 180 degrees about A_1. This also works if the same rotation is performed about A_2. The translations in the y direction are similar. If the planar surface is tiled with "strings," translations in the x direction are not possible. Translations in the y direction are done with a rotation of 90 degrees about A_1 followed by a rotation of -90 degrees about A_2.

To decide which gait is most appropriate for a locomotion task, we introduce a gait complexity measure. The complexity of a gait with respect to a unit translation[4] is computed as the number and size of the distinct rotations required to do the motion.

We can combine the gait complexity measure with the reachability results in a decision procedure for choosing the appropriate underlying tiling for particular locomotion tasks. If locomotion is required along arbitrary directions, pair tilings is most appropriate. If locomotion is required along orthogonal directions, the ring tiling is most appropriate. If locomotion is required along one direction only, the string tiling is most appropriate. Pair and ring tilings are also possible, but the string tiling is the most efficient. The string tiling can be parallelized so that we can imagine a locomotion modality where such a structure could be "rolled forward".

8 Experiments

We have built one Molecule and implemented the control algorithms for linear motion, convex transition, and concave transition. Because we only have one Molecule, we simulated a lattice of Molecules by building a plexiglass structure that supports linear translations as well as convex and concave transitions. We bolted inter-Molecule connectors at the locations where we expect to have Molecules. We performed experiments in which the Molecule successfully performed linear translations and convex and concave transitions. The Molecule used dead-reckoning to locate the connectors.

The errors did not seem to accumulate from step to step. We believe this is because after each individual rotation the Molecule connects with a rigid connection. The connection sheaths aligned the Molecules, minimizing the orientation errors.

9 Discussion

Our vision is to create robots by using self-reconfiguration: hundreds of small modules can autonomously organize and reorganize as geometric structures to best fit the terrain on which the robot has to move or the shape the object the robot has to manipulate. In this paper we described our design for a module we call a Molecule that can be used to aggregate as arbitrary three-dimensional self-reconfigurable structures. We described the structure of the Molecule, the control algorithms for effecting linear translations, convex transitions, and concave transitions, and discuss the potential for this Molecule to be a universal module. We discussed geometric trajectory control algorithms for planning the motion of an individual Molecule on a substrate of other Molecules. We then showed the geometric construction of a graph that reduces the Molecule trajectory planning to a graph search. Finally, we examined gaits for three different structures of the underlying Molecule substrate. An expression for the final trajectory in the rotation space of the Molecule can be extracted automatically from this graph.

Much work on global motion planning algorithms for Molecular structures needs to be done. We are currently investigating several approaches to defining gaits for moving Molecular structures of different terrain types, and transformations between these gaits.

Our current experiments should we viewed as pilot studies for the Molecular design. These experiments have established that it is possible to control the four degrees of freedom of Molecules to effect rotations of $+90$ degrees and -90 degrees. Individual Molecules can be controlled to move relative to other Molecules to effect linear translations and convex and concave

[4] Unit translations can be translations in the x direction by one molecular unit, translations in the y direction by one molecular unit, or any user-defined measure that is relevant to the required locomotion task.

transitions. These results are promissing. We plan to build 16 Molecules to evaluate our self-reconfiguring theory.

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